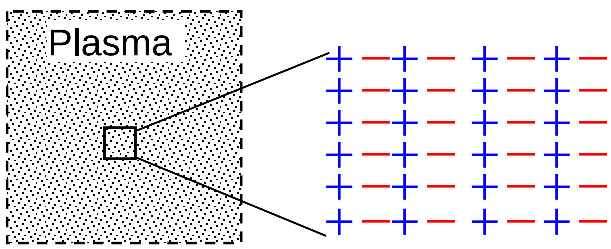
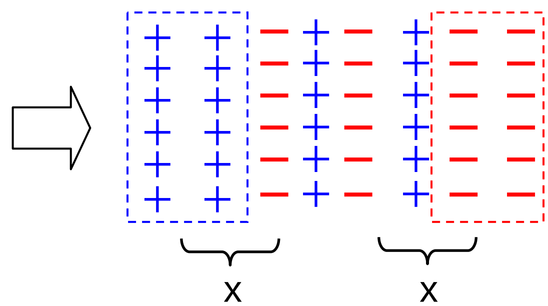
29:129 – Plasma Oscillations— An application of electrostatics and classical mechanics

We consider a gas of electrons and positive ions (plasma). The plasma is overall neutral, i.e., the number density of the electrons and ions are the same. We think of the plasma as two interpenetrating fluids — an negative electron fluid and a positive ion fluid.



Schematic diagram of the plasma, with the inset showing a typical volume within the plasma with equal densities of positive ions and electrons.



Small volume of plasma in which the Electrons are displaced to the right by an amount x, while the ions are fixed.

Under normal conditions, there are always equal numbers positive ions and electrons in any volume of the plasma, so the charge density ρ = 0, and there is no large scale electric field in the plasma. Now imagine that all of the electrons are displaced to the right by a small amount x, while the positive ions are held fixed, as shown on the right side of the figure above.

The displacement of the electrons to the right leaves an excess of positive charge on the left side of the plasma slab and an excess of negative charge on the right side, as indicated by the dashed rectangular boxes. The positive slab on the left and the negative slab on the right produce an electric field pointing toward the right that pulls the electrons back toward their original locations. However, the electric force on the electrons causes them to accelerate and gain kinetic energy, so they will overshoot their original positions. This situation is similar to a mass on a horizontal frictionless surface connected to a horizontal spring. The spring provides a restoring force that always acts to bring the mass back to its equilibrium position, thus producing simple harmonic motion. In the present problem, the electrons execute simple harmonic motion at a frequency that is called the electron plasma frequency, $f_{pe} = \omega_{pe}/2\pi$, where ω_{pe} is the angular plasma frequency. We can apply what we know from electrostatics to compute the electric field that acts on the electrons, and use this in Newton's second law to obtain ω_{pe}. Let n_e be the number density (in m^{-3}) of the electrons in the plasma, each having a charge of magnitude e. We can think of the positive region on the left side and the negative region on the right side as forming two parallel charged planes, each having a cross sectional area A and width x. The electric field between the two charge slabs can be computed using Gauss's law and is given by

$$E = \frac{Q}{\varepsilon_o A}, \tag{1}$$

where Q is the total charge in the slabs, which can be written in terms the charge density ρ and the volume of the slab, V as $Q = \rho V = (en_e)(Ax)$, so that the electric field in (1) is

$$E = \frac{en_e Ax}{\varepsilon_o A} = \frac{en_e x}{\varepsilon_o}. \tag{2}$$

The force on the electrons is $-eE$, and using this in Newton's second law we get

$$m_e \frac{d^2x}{dt^2} = -e \cdot \frac{en_e}{\varepsilon_o} x = -\frac{e^2 n_e}{\varepsilon_o} x, \tag{3}$$

where m_e is the electron mass. Equation (3) can be put in the standard form for a simple harmonic oscillator

$$\frac{d^2x}{dt^2} + \frac{e^2 n_e}{\varepsilon_o m_e} x = 0. \tag{4}$$

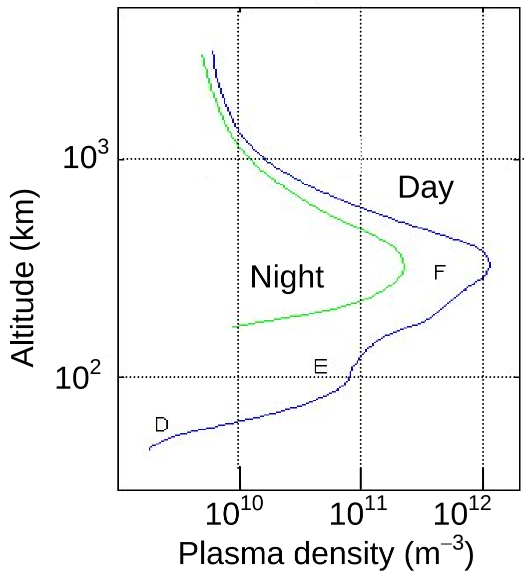
The equation for a simple harmonic oscillator is of the form $\ddot{x} + \omega^2 x = 0$, where ω is the angular frequency of oscillation. Thus we identify the *frequency of electron plasma oscillations* in (4) as

$$\omega_{pe} = 2\pi f_{pe} = \sqrt{\frac{e^2 n_e}{\varepsilon_o m_e}}. \tag{5}$$

We will calculate the value of $f_{pe} = \omega_{pe}/2\pi$ for a few cases of interest. Note that f_{pe} depends only on the density of the electrons, all the other factors are constants.

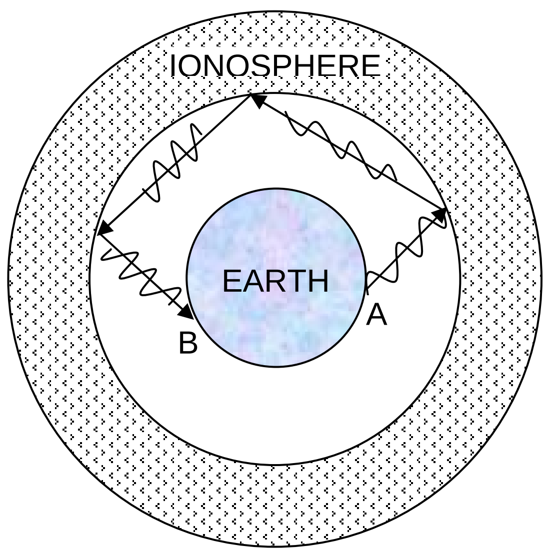
(a) *A typical laboratory plasma*.—The plasmas down in the plasma lab, B01 VAN have an electron density ~ 10^{15} m^{-3}, so that $f_{pe} \approx 300$ MHz.

(b) *The ionosphere*.—The upper layer of the Earth's atmosphere, above an altitude of roughly 90 km) that is ionized by solar UV radiation. The density in the ionosphere varies with altitude and also there is a strong diurnal (day/night) variation. If we take an electron density of 1×10^{11} m^{-3}, then, $f_{pe} \approx 3$ MHz. This means that any radio waves below 3 MHz will be reflected by the ionosphere; in other words, the ionosphere is opaque to radio waves below f_{pe} and transparent to waves above f_{pe}. To communicate with satellites in low earth orbit (LEO – up to about 2000 km altitude), frequencies above f_{pe} must be used.



Altitude profile of ionospheric plasma density. The ionosphere is divided into regions, D, E, F, with the F region having the highest plasma density. Plasma begins to appear in the atmosphere at an altitude of about 85 km. The plasma up to a few thousand km is weakly ionized; it contains more neutral atoms and molecules than ions and electrons.

The reflection of radio waves below f_{pe} allows transmission from one point on the earth to another by multiple ionospheric reflections.This allows for communications between points A and B on the earth that are not in the line of sight.



Multiple reflections of radio waves from the ionosphere, allowing for communication between points A and B.

Ionospheric conditions are severely affected by solar flares and CMEs (coronal mass ejections). The charged particles from the sun eventually travel to earth where some get trapped by the earth's magnetic field. When these particles get down to ionospheric altitudes (space physicists call this precipitation) they produce substorms, which are disturbances in the ionospheric plasma. Some of the particles which precipitate down in the high latitude regions, get accelerated and produce the aurora.

(c) *Metals.*—The concept of electron plasma oscillations can also be applied to the free electrons in a conductor. For example, the free electron density in Cu is 8.4×10^{28} m^{-3}. (By way of comparison, the density of air molecules at a pressure of one atmosphere and T = 300K is 2.4×10^{25} m^{-3}.) In Cu, then $f_{pe} \approx 2.6\times10^{15}$ Hz. This is higher than the frequencies of visible light, and explains why metals are opaque to visible light. The light energy is absorbed by the conduction electrons which oscillate and transfer some of this energy to the ionic lattice